

Quantum dissipative effects in moving imperfect mirrors: sidewise and normal motions

César D. Fosco^{1,2} *, Fernando C. Lombardo³ †, and Francisco D. Mazzitelli^{1,3} ‡

¹ *Centro Atómico Bariloche Comisión Nacional de Energía Atómica, R8402AGP Bariloche, Argentina*

² *Instituto Balseiro, Comisión Nacional de Energía Atómica, R8402AGP Bariloche, Argentina and*

³ *Departamento de Física Juan José Giambiagi,
FCEyN UBA, Facultad de Ciencias Exactas y Naturales,
Ciudad Universitaria, Pabellón I, 1428 Buenos Aires, Argentina*

(Dated: today)

We extend our previous work on the functional approach to the dynamical Casimir effect, to compute dissipative effects due to the relative motion of two flat, parallel, imperfect mirrors in vacuum. The interaction between the internal degrees of freedom of the mirrors and the vacuum field is modeled with a nonlocal term in the vacuum field action. We consider two different situations: either the motion is ‘normal’, i.e., the mirrors advance or recede changing the distance $a(t)$ between them; or it is ‘parallel’, namely, a remains constant, but there is a relative sliding motion of the mirrors’ planes. For the latter, we show explicitly that there is a non-vanishing frictional force, even for a constant shifting speed.

PACS numbers:

I. INTRODUCTION

Interesting manifestations of the vacuum electromagnetic field fluctuations may arise when a neutral body (‘mirror’) is subjected to the influence of certain time-dependent external conditions. A nice particular example of this kind of phenomenon occurs when those varying conditions amount to a *motion* of the body. When this body is an accelerated mirror, this is the celebrated dynamical Casimir effect (DCE), or ‘motion induced radiation’, whereby real photons are created out of the vacuum. As with any radiation phenomenon, it can be described from at least two points of view: for the external agent driving the body, this process is perceived as the cause of a dissipative force (which reacts against the change in the external conditions), while, on the other hand, an observer measuring electromagnetic field properties detects creation of real photons out of the vacuum.

Under the currently accessible experimental conditions, both the dissipative force and the number of photons that may be created by a single accelerated mirror are deceptively small. The main reason is that (for an oscillatory motion) it would be necessary to attain prohibitively high mechanical frequencies for the effect to be detected. There are, however, some experimental setups where the effect may be purposely enhanced; for example, when a moving mirror is part of an electromagnetic cavity, the mechanical oscillations of the mirror may resonate with the corresponding normal cavity modes. In that case, parametric amplification produces an exponential growth in the number of emitted photons, at least for the idealized case of ‘perfect’ mirrors, namely, ones that behave as ideal conductors. For this kind of configuration, the estimated effect is much closer to experimental verification and there are, indeed, several ongoing experiments with that goal in mind.

On the other hand, some alternative proposals invoke the use of time-dependent changes in the electromagnetic *properties* of the body, whereby mechanical motion is altogether avoided. For general reviews of these, and other aspects of the dynamical Casimir effect, see, for example [1].

An interesting particular case of motion induced effects is the so-called ‘quantum friction’, also due to the vacuum electromagnetic fluctuations, where the theory predicts the appearance of non-contact frictional forces between neutral bodies in relative sidewise motion. This effect can be understood in terms of the interchange of virtual photons between the bodies, that then produce excitations of their internal degrees of freedom. This quantum friction has been analyzed [2] (and debated [3]) at length, mainly for the case of half-spaces shifting with constant velocity (see also Refs. in [4], where two atoms on parallel trajectories, or an atom moving on a half-space are considered as a first approach to the more general situation of emission of light from sheared dielectric surfaces).

Therefore, dissipative effects on moving bodies may not only be produced by the excitation of real photons out of the quantum vacuum, but also of the mirror’s internal degrees of freedom, by the mediation of the vacuum field (in this case by virtual photons). Note, however, that there is another source of quantum dissipation, independent of

* fosco@cab.cnea.gov.ar

† lombardo@df.uba.ar

‡ fmazzi@df.uba.ar

the coupling to the electromagnetic field, which is the excitation of the internal degrees of freedom due to ‘fictitious’, inertial forces [5] in an accelerated system.

In a previous paper [6] we used a functional approach to study the DCE for a single zero-width mirror, in a model where the interaction between the internal degrees of freedom and the vacuum field was described by a local δ -potential term in the vacuum field action, which had support on the (time-dependent) mirror’s position. Although sufficient to explain the production of real photons out of the vacuum, this model has no room to describe processes where the mirror’s degrees of freedom are excited. In that respect, one must consider more general models. For example, for zero-width mirrors, the model should include a *nonlocal* term quadratic in the vacuum field. Indeed, this is the kind of term one naturally derives by integrating out microscopic degrees of freedom in some models [7]. But one could also argue, on more general grounds, that a (space or time) nonlocality must exist (in a zero-width term) since most microscopic models shall indeed produce a nontrivial momentum dependence in the interaction term, which is seen as a certain nonlocality in spacetime.

With the purpose to overcome that limitation in mind, in this paper we analyze dissipative effects on moving mirrors, using a functional formalism, generalizing our previous results [6] in several directions. First, we consider nonlocal boundary interaction terms in the vacuum field action, to allow for a better description of the effects of the microscopic degrees of freedom. We do that for either one or two flat mirrors, both for parallel and normal motion. This will allow us to describe, on the same footing, dissipative forces coming from vacuum field or matter (microscopic) degrees of freedom. In particular, we shall show that non-contact frictional forces do appear between thin mirrors, even for parallel motion at a constant speed. Then, quantum friction induced by the sideways motion between dielectric mirrors is obtained as a byproduct of the effective action for the mirror coordinates. This effective action is the result of tracing out quantum fluctuations of the vacuum field and the internal degrees of freedom in the plates.

The structure of this paper is as follows: in II we introduce the model which is used in the rest of the paper as a convenient prototype to study the effects mentioned above: it consists of a quantum scalar field coupled to the microscopic degrees of freedom of one or two imperfect flat mirrors. The corresponding effective action due to the mirrors can be formally written in terms of a functional determinant, which in turn depends on the scalar field (via a function of its free propagator) and also on kernels that encode the properties of the mirrors.

In Section III, the general formal expression for the effective action is evaluated perturbatively for two mirrors in a normal motion situation. We present some explicit calculations in $1+1$ dimensions for perfect and imperfect mirrors, showing that the imaginary part of the effective action peaks when the motion of the mirrors is in resonance with the standing waves of the cavity. In Section IV, we compute the effective action for parallel motion. We first show that, for the interaction term to describe a zero width mirror where the vacuum field propagates at a different speed than in the vacuum, such interaction term must necessarily be nonlocal. Then we show that, in that situation, nontrivial dissipative effects appear, even for parallel motion with constant velocity. We also present the perturbative form of the effective action for non-constant parallel velocity of one of the mirrors. Section V contains our conclusions.

II. IMPERFECT ZERO-WIDTH MIRRORS COUPLED TO A REAL SCALAR FIELD

A. One mirror

Let us first consider a single, flat, non-relativistic mirror, coupled to a real scalar field φ , in $3+1$ dimensions. We shall setup our conventions for Euclidean (imaginary time) spacetime, although we shall occasionally undo the Wick’s rotation to calculate the imaginary part of the real time effective action. Coordinates shall be denoted by x_μ ($\mu = 0, 1, 2, 3$), and the reference system is chosen in such a way that, at any given time, the mirror occupies an $x_3 = \text{constant}$ plane, where the constant will generally be time-dependent. Besides, the coordinates on those planes are denoted by $x_\parallel \equiv (x_1, x_2)$. Note that, under purely parallel motion, the plane is invariant, but the coordinates x_\parallel of the points of the mirror will change. We shall only consider cases where the sliding motion has a constant direction, thus, one can add to the choice of reference system the extra requirement that only the x_1 coordinate of points on the mirror can carry a time dependence. Thus, (purely) normal motion of the mirror will be described by a single function of time, $x_3 = q_\perp(x_0)$, say; a single function is also sufficient for purely parallel motion, we shall denote it by $x_1 = q_\parallel(x_0)$.

The vacuum field φ is coupled to the internal degrees of freedom of the mirror, that we will denote generically by ψ , living on the $2+1$ dimensional spacetime swept by the mirror in its time evolution. The effective action Γ for the mirror coordinates, due to the quantum fluctuations of both φ and ψ , will be a functional of q_\perp and q_\parallel . It may be explicitly written, in terms of an Euclidean functional integral:

$$e^{-\Gamma(q_\perp, q_\parallel)} = \int \mathcal{D}\varphi \mathcal{D}\psi e^{-S_0(\varphi) - S_m^{(0)}(\psi) - S_m^{(\text{int})}(\varphi, \psi)}, \quad (1)$$

where S_0 is the action of the vacuum field in the absence of the mirror; which corresponds to a free real scalar field:

$$S_0 = \frac{1}{2} \int d^4x [(\partial\varphi)^2 + m^2\varphi^2] . \quad (2)$$

On the other hand, $S_m^{(0)}$ denotes the free action for the microscopic degrees of freedom, while $S_m^{(\text{int})}$ is the term that couples them to the vacuum field.

A convenient approach to find an expression for the effective action Γ is to perform the integration of the two relevant fields in two successive steps. Considering first the microscopic degrees of freedom on the mirrors, and under the usual assumption that the medium behaves linearly, one keeps just the quadratic term in the result of that integral,

$$e^{-\Gamma(q_\perp, q_\parallel)} = \int \mathcal{D}\varphi e^{-S_0(\varphi) - S_I(\varphi)} , \quad (3)$$

where

$$S_I(\varphi) = \frac{1}{2} \int d^4x d^4x' \varphi(x') V(x, x') \varphi(x) . \quad (4)$$

In (3) we have neglected a term which, albeit independent of φ , may, for a non-rigid motion of the mirror, account for the excitation of internal degrees of freedom due to the acceleration of the mirror. These non inertial effects, described in [5], will not be considered in the present work since we are here interested in effects where the vacuum field fluctuations are relevant.

Regarding the form of $V(x, x')$, since both $S_m^{(0)}$ and $S_m^{(\text{int})}$ are localized on the mirror's world-volume,

$$V(x, x') = \delta(x_3 - q_\perp(x_0)) \Lambda(x_0, x_\parallel; x'_0, x'_\parallel) \delta(x'_3 - q_\perp(x'_0)) , \quad (5)$$

where Λ is a -yet unspecified- function, which, *in a system where the mirror has no parallel motion* is assumed to be nonlocal only in time, namely,

$$\Lambda(x_0, x_\parallel; x'_0, x'_\parallel) = \lambda(x_0 - x'_0) \delta^{(2)}(x_\parallel - x'_\parallel) . \quad (6)$$

The simplifying assumption that $\Lambda(x_0, x_\parallel; x'_0, x'_\parallel)$ is only nonlocal in time is tantamount to assuming that its reflection and transmission coefficients shall be only a function of frequency, and not on the parallel component of the momentum of the incident scalar field waves. In other words, the fluctuations of ψ are only time dependent, they do not propagate in the parallel direction.

The fact that Λ adopts its simplest form in a *comoving* system deserves perhaps a little extra clarification: Λ describes the correlation *mediated by the microscopic field*, of two φ -field fluctuations localized on the world-volume of the mirror. In a comoving system, that correlation will, since the properties of the medium are assumed to be independent of time, depend only on the difference between the times.

As a concrete example, in the comoving system one may consider a mirror at $x_3 = 0$, composed of decoupled one-dimensional oscillators at each point, $Q(x_\parallel, x_0)$, each one coupled to φ at its respective position, that is

$$\begin{aligned} S_m^{(0)} &= \frac{1}{2} \int dx_0 \int d^2x_\parallel \left[\dot{Q}(x_\parallel, x_0)^2 + \Omega^2 Q(x_\parallel, x_0)^2 \right] \\ S_m^{\text{int}} &= ig \int d^4x Q(x_\parallel, x_0) \delta(x_3) \varphi(x_0, x_\parallel, x_3) , \end{aligned} \quad (7)$$

where Ω and g are positive constants. It is straightforward to check that, in this case, the integration over the internal degrees of freedom produces an interaction term S_I with a two-point function $\Lambda(x_0, x_\parallel; x'_0, x'_\parallel)$ of the form given in Eq.(6), with:

$$\lambda(x_0 - x'_0) = \frac{g^2}{2\Omega} e^{-\Omega|x_0 - x'_0|} . \quad (8)$$

In the limit where the oscillators become extremely rigid, $\Omega \rightarrow \infty$, one obtains a time-local interaction:

$$\lambda(x_0 - x'_0) \rightarrow \left(\frac{g}{\Omega}\right)^2 \delta(x_0 - x'_0) . \quad (9)$$

On the other hand, as seen from the Lab system, when the mirror has parallel motion, the situation does change: Λ may depend on q_\parallel , since the microscopic degrees of freedom are not at rest, and their collective motion may contribute

to the correlation of vacuum field fluctuations between points with a spatial separation. The relevant modifications one should implement for parallel motion are discussed in Section IV. Since the speeds are assumed to be non-relativistic we shall neglect, for both parallel and normal motions, the time dilation effects that could also affect the form of λ .

Coming back to the construction of the model, the final step is to integrate out the vacuum field itself,

$$\Gamma(q_{\perp}, q_{\parallel}) = \frac{1}{2} \log \det(-\partial^2 + V) = \frac{1}{2} \text{Tr} \log(-\partial^2 + V). \quad (10)$$

When rotated back to Minkowski spacetime, this effective action contains information about the reaction force exerted by the vacuum on the moving mirror, and also on the probabilities of exciting anyone of the two fields that have been integrated out.

B. Two mirrors

The previous considerations can be generalized to several moving mirrors in a rather straightforward way. We are here interested in case of just two mirrors. Besides, since only their relative motion may affect the physical results, we shall use a reference system, the laboratory frame, where one of them is at rest. In our conventions, that mirror will be the ‘left’ (L) one, located at $x_3 = 0$, while the other, ‘right’ (R) mirror, shall move rigidly in either the normal or parallel direction.

The effective action is, in this case, given by an expression that generalizes (10):

$$\Gamma(q_L, q_R) = \frac{1}{2} \log \det(-\partial^2 + V_L + V_R) = \frac{1}{2} \text{Tr} \log(-\partial^2 + V_L + V_R), \quad (11)$$

where V_L (V_R) is the kernel for the interaction between the vacuum field φ and the left (right) mirror. We denote by q_L (q_R) the functions that describe either the normal or parallel motion of the left (right) mirror.

This effective action may be decomposed into three terms which make it easier to disentangle the role of each mirror’s individual motion from the contribution of their relative motion. Indeed, defining $\Gamma_I(q_L, q_R)$ by

$$\Gamma(q_L, q_R) = \Gamma(q_L) + \Gamma(q_R) + \Gamma_I(q_L, q_R), \quad (12)$$

where the first two terms are just the effective actions corresponding to each individual mirror (in the absence of the other), while $\Gamma_I(q_L, q_R)$, by its very definition, captures the part of the effective action which depends on the influence between the two mirrors.

In some particular situations it is unnecessary to perform the subtraction above to find Γ_I . Indeed, if one knows, on physical grounds, that each mirror by itself has a trivial effective action, Γ equals (except for irrelevant constants) Γ_I . This shall be the case for parallel motion with constant speed.

In the next two sections, we evaluate Γ_I for two different situations, purely normal or purely parallel motion, within the context of different approximations.

III. NORMAL MOTION

Under the assumption that the L mirror is static (in the chosen reference system) and that there is only normal motion for the R mirror: $q_{\perp} \neq 0$ and $q_{\parallel} = 0$, the kernels for V_L and V_R become:

$$V_A(x, x') = \delta(x_3 - q_A(x_0)) \lambda(x_0 - x'_0) \delta(x'_3 - q_A(x'_0)) \delta^{(2)}(x_{\parallel} - x'_{\parallel}), \quad (13)$$

where $A = L, R$, with $q_L = 0$, $q_R = q_{\perp} \neq 0$.

In this case, an expression for Γ_I may be obtained, for example, by a natural extension of the auxiliary field method used in [8] for the static Casimir effect:

$$\Gamma_I(q_L, q_R) = \Gamma_I(q_{\perp}) = \frac{1}{2} \log \det(\mathcal{K}) = \frac{1}{2} \text{Tr}(\log \mathcal{K}), \quad (14)$$

where \mathcal{K} is a 2×2 matrix of kernels, $\mathcal{K}_{AB}(x_0, x_{\parallel}; x'_0, x'_{\parallel})$,

$$\mathcal{K}_{AB}(x_0, x_{\parallel}; x'_0, x'_{\parallel}) = \Delta[x_0, x_{\parallel}, q_A(x_0); x'_0, x'_{\parallel}, q_B(x'_0)] + \lambda^{-1}(x_0 - x'_0) \delta^{(2)}(x_{\parallel} - x'_{\parallel}) \delta_{AB}, \quad (15)$$

where Δ is the free scalar field propagator in coordinate space and $\lambda^{-1}(x_0 - x'_0)$ is the inverse of $\lambda(x_0 - x'_0)$, defined by

$$\int dx''_0 \lambda(x_0 - x''_0) \lambda^{-1}(x''_0 - x'_0) = \delta(x_0 - x'_0). \quad (16)$$

Symmetry under translations in the parallel directions may be exploited to perform a Fourier transform in those coordinates, so that the problem is in fact essentially one-dimensional:

$$\Gamma_I(q_L, q_R) = \frac{\Sigma}{2} \int \frac{d^2 k_{\parallel}}{(2\pi)^2} \text{Tr}(\log \tilde{\mathcal{K}}), \quad (17)$$

where Σ is the area of the mirrors and

$$\tilde{\mathcal{K}}_{AB}(x_0, x'_0; k_{\parallel}) = \int d^2 x_{\parallel} e^{-ik_{\parallel} \cdot (x_{\parallel} - x'_{\parallel})} \mathcal{K}_{AB}(x_0, x'_0; x_{\parallel} - x'_{\parallel}), \quad (18)$$

where we have made explicit the fact the kernel can only depend on differences of parallel coordinates. Thus,

$$\tilde{\mathcal{K}}_{AB}(x_0, x'_0; k_{\parallel}) = \tilde{\Delta}[x_0, q_A(x_0); x'_0, q_B(x'_0); k_{\parallel}] + \lambda^{-1}(x_0 - x'_0) \delta_{AB}. \quad (19)$$

Note that the trace affects now the time-coordinate continuous indices and the discrete space of L and R components, but a partial trace over the parallel space is implemented by the momentum integral. In the rest of this section we will omit the factor Σ , i.e. we will compute the effective actions and forces per unit area of the mirrors.

Following [6], we compute the effective action assuming small displacements of the moving mirror around its equilibrium position. Expanding the matrix elements $\tilde{\mathcal{K}}_{AB}$ in powers of q_{\perp} ,

$$\tilde{\mathcal{K}} = \tilde{\mathcal{K}}_0 + \tilde{\mathcal{K}}_1 + \tilde{\mathcal{K}}_2 + \dots \quad (20)$$

where the subindices denote the order of each term (in order to simplify the notation we omitted the A, B labels of each matrix element). Therefore,

$$\Gamma_I(q_{\perp}) = \frac{1}{2} \int \frac{d^2 k_{\parallel}}{(2\pi)^2} \text{Tr} \log [1 + \tilde{\mathcal{K}}_0^{-1}(\tilde{\mathcal{K}}_1 + \tilde{\mathcal{K}}_2 + \dots)], \quad (21)$$

where we omitted a divergent term independent of the position of the mirrors. Keeping up to quadratic terms in $q_{\perp}(\tau)$ we have

$$\Gamma_I(q_{\perp}) = \frac{1}{2} \int \frac{d^2 k_{\parallel}}{(2\pi)^2} \text{Tr} [\tilde{\mathcal{K}}_0^{-1}(\tilde{\mathcal{K}}_1 + \tilde{\mathcal{K}}_2) - \frac{1}{2}(\tilde{\mathcal{K}}_0^{-1}\tilde{\mathcal{K}}_1)^2]. \quad (22)$$

We now present some details of the evaluation of Γ_I , and of some physical observables derived from it. We do that mostly in 3 + 1 dimensions; the special case of 1 + 1 dimensions is considered at the end.

First we consider the form of the kernels $\tilde{\mathcal{K}}_{iAB}$, $i = 0, 1, 2$. From (19), we see that the matrix elements of the operator $\tilde{\mathcal{K}}$ read, to the lowest order,

$$\tilde{\mathcal{K}}_{0AB}(x_0, x'_0; k_{\parallel}) = \int \frac{d\omega}{2\pi} e^{i\omega(x_0 - x'_0)} \tilde{\tilde{\mathcal{K}}}_{0AB}(\omega, k_{\parallel}), \quad (23)$$

where

$$\tilde{\tilde{\mathcal{K}}}_{0AB}(\omega, k_{\parallel}) = \frac{1}{\tilde{\lambda}(\omega)} \delta_{AB} + \tilde{\Delta}_{0AB}(\omega, k_{\parallel}) \quad (24)$$

and

$$\tilde{\Delta}_0(\omega, k_{\parallel}) = \frac{1}{2\sqrt{\omega^2 + k_{\parallel}^2 + m^2}} \begin{pmatrix} 1 & e^{-a\sqrt{\omega^2 + k_{\parallel}^2 + m^2}} \\ e^{-a\sqrt{\omega^2 + k_{\parallel}^2 + m^2}} & 1 \end{pmatrix}. \quad (25)$$

In the above equations we have introduced the Fourier transform

$$\tilde{\lambda}(\omega) = \int d\tau e^{-i\omega\tau} \lambda(\tau). \quad (26)$$

To first order in $q_\perp(x_0)$ we have

$$\tilde{\mathcal{K}}_{1LR}(x_0, x'_0; k_\parallel) = \tilde{\mathcal{K}}_{1RL}(x'_0, x_0; k_\parallel) = -\frac{q_\perp(x_0)}{2} \int \frac{d\omega}{2\pi} e^{i\omega(x_0-x'_0)} e^{-a\sqrt{\omega^2+k_\parallel^2+m^2}}, \quad (27)$$

while $\tilde{\mathcal{K}}_{1LL} = \tilde{\mathcal{K}}_{1RR} = 0$.

The second order non-vanishing matrix elements are

$$\begin{aligned} \tilde{\mathcal{K}}_{2RR}(x_0, x'_0, k_\parallel) &= \frac{1}{4} [q_\perp(x_0) - q_\perp(x'_0)]^2 \int \frac{d\omega}{2\pi} e^{i\omega(x_0-x'_0)} \sqrt{\omega^2 + k_\parallel^2 + m^2} \\ \mathcal{K}_{2LR}(x_0, x'_0, k_\parallel) &= \frac{1}{4} q_\perp(x_0)^2 \int \frac{d\omega}{2\pi} e^{i\omega(x_0-x'_0)} e^{-a\sqrt{\omega^2+k_\parallel^2+m^2}} \sqrt{\omega^2 + k_\parallel^2 + m^2} \end{aligned} \quad (28)$$

and $\tilde{\mathcal{K}}_{2RL}(x'_0, x_0; k_\parallel) = \tilde{\mathcal{K}}_{2LR}(x_0, x'_0; k_\parallel)$.

A. Linear term in q : the static Casimir force

As a first result, we obtain the form of the linear term in the effective action. It is given by

$$\begin{aligned} \Gamma_1(q_\perp) &= \frac{1}{2} \int \frac{d^2 k_\parallel}{(2\pi)^2} \text{Tr}[\tilde{\mathcal{K}}_0^{-1} \tilde{\mathcal{K}}_1] = \frac{1}{2} \int \frac{d^2 k_\parallel}{(2\pi)^2} \int dx_0 dx'_0 \left[\tilde{\mathcal{K}}_{0LR}^{-1}(x_0, x'_0, k_\parallel) \tilde{\mathcal{K}}_{1RL}(x_0, x'_0; k_\parallel) \right. \\ &\quad \left. + \tilde{\mathcal{K}}_{0RL}^{-1}(x_0, x'_0; k_\parallel) \tilde{\mathcal{K}}_{1LR}(x_0, x'_0; k_\parallel) \right]. \end{aligned} \quad (29)$$

The inverse of the free kernel, $\tilde{\mathcal{K}}_0^{-1}$, can be exactly evaluated because $\tilde{\mathcal{K}}_0$ is diagonal in frequency space. Its Fourier transform is given by

$$\tilde{\mathcal{K}}_0^{-1}(\omega, k_\parallel) = \frac{1}{c^2(\omega, k_\parallel) - b^2(\omega, k_\parallel)} \begin{pmatrix} c(\omega, k_\parallel) & -b(\omega, k_\parallel) \\ -b(\omega, k_\parallel) & c(\omega, k_\parallel) \end{pmatrix}, \quad (30)$$

where we have introduced:

$$\begin{aligned} b(\omega, k_\parallel) &= \frac{e^{-a\sqrt{\omega^2+k_\parallel^2+m^2}}}{2\sqrt{\omega^2+k_\parallel^2+m^2}} \\ c(\omega, k_\parallel) &= \frac{1}{\tilde{\lambda}(\omega)} + \frac{1}{2\sqrt{\omega^2+k_\parallel^2+m^2}}. \end{aligned} \quad (31)$$

Therefore, we obtain for the linear part of the effective action

$$\Gamma_1(q_\perp) = -\frac{1}{2} \int dx_0 q_\perp(x_0) \int \frac{d\omega}{2\pi} \int \frac{d^2 k_\parallel}{(2\pi)^2} \frac{1}{\sqrt{\omega^2+k_\parallel^2+m^2}} \frac{e^{-2a\sqrt{\omega^2+k_\parallel^2+m^2}}}{\left(\frac{1}{\tilde{\lambda}(\omega)} + \frac{1}{2\sqrt{\omega^2+k_\parallel^2+m^2}} \right)^2 - \frac{e^{-2a\sqrt{\omega^2+k_\parallel^2+m^2}}}{4(\omega^2+k_\parallel^2+m^2)}}. \quad (32)$$

This result has a clear interpretation. The functional variation of the effective action with respect to $q_\perp(x_0)$ gives the force on the left mirror, which in this approximation is time independent and given by the usual static Casimir force between thin mirrors characterized by a function $\tilde{\lambda}(\omega)$. The usual Casimir force for Dirichlet mirrors is recovered in the $m = 0$ and $\tilde{\lambda} = \infty$ limit.

B. Quadratic term: decay of the vacuum and dissipative force on the moving mirror

The linear term in the effective action does not contain information about the back-reaction of the created particles on the motion of the mirror. Indeed, we have seen in the previous subsection that it has the form

$$\Gamma_1(q_\perp) = \int dx_0 q_\perp(x_0) F_C \quad (33)$$

where F_C is the (time-independent) static Casimir force between imperfect mirrors separated by a distance a . On general grounds, we expect that, to second order,

$$\Gamma_2(q_\perp) = \frac{1}{2} \int dx_0 \int dx'_0 q_\perp(x_0) F(x_0 - x'_0) q_\perp(x'_0). \quad (34)$$

This nonlocal effective action contains (in its kernel) information on both the dissipative force on the mirror and on the probability of creating φ excitations.

To calculate F , we note that, from (17) and (22):

$$\Gamma_2(q_\perp) = \frac{1}{2} \int \frac{d^2 k_\parallel}{(2\pi)^2} \text{Tr} [\tilde{\mathcal{K}}_0^{-1} \tilde{\mathcal{K}}_2 - \frac{1}{2} (\tilde{\mathcal{K}}_0^{-1} \tilde{\mathcal{K}}_1)^2] \equiv \Gamma_2^{(1)} + \Gamma_2^{(2)}. \quad (35)$$

The first contribution can be written more explicitly as

$$\Gamma_2^{(1)} = \frac{1}{2} \int dx_0 \int dx'_0 \int \frac{d^2 k_\parallel}{(2\pi)^2} \tilde{\mathcal{K}}_{0AB}^{-1}(x_0, x'_0; k_\parallel) \tilde{\mathcal{K}}_{2BA}(x'_0, x_0; k_\parallel) \quad (36)$$

The term proportional to $\tilde{\mathcal{K}}_{0RR}^{-1} \tilde{\mathcal{K}}_{2RR}$ carries a dependence in a , and it describes, when $a \rightarrow \infty$, the effective action for the moving R mirror in the absence of the L mirror: we already computed this single-mirror term in [6], for the particular case of a constant $\tilde{\lambda}$.

For a finite a , and after discarding terms which renormalize the mass of the mirror, we see that:

$$\Gamma_2^{(1)} = \frac{1}{2} \int dx_0 \int dx'_0 q_\perp(x_0) F^{(1)}(x_0 - x'_0) q_\perp(x'_0) \quad (37)$$

where the Fourier transform of $F^{(1)}$ is

$$\widetilde{F^{(1)}}(\omega) = -\frac{1}{4\pi} \int d\nu \int \frac{d^2 k_\parallel}{(2\pi)^2} \frac{c(\nu + \omega) \sqrt{\nu^2 + k_\parallel^2 + m^2}}{c^2(\nu + \omega, k_\parallel) - b^2(\nu + \omega, k_\parallel)}. \quad (38)$$

On the other hand, the terms proportional to $\tilde{\mathcal{K}}_{0LR}^{-1} \tilde{\mathcal{K}}_{2RL}$ and $\tilde{\mathcal{K}}_{0RL}^{-1} \tilde{\mathcal{K}}_{2LR}$ are of the form $\Delta M^2 \int dx_0 q^2(x_0)$ and produce a (finite) renormalization of the mass of the mirror, which we again discard from our expression.

We now evaluate the second contribution to (35), which reads

$$\Gamma_2^{(2)}(q_\perp) = -\frac{1}{4} \int d\tau_1 d\tau_2 d\tau_3 d\tau_4 \tilde{\mathcal{K}}_{0AB}^{-1}(\tau_1, \tau_2; k_\parallel) \tilde{\mathcal{K}}_{1BC}(\tau_2, \tau_3; k_\parallel) \tilde{\mathcal{K}}_{0CD}^{-1}(\tau_3, \tau_4; k_\parallel) \tilde{\mathcal{K}}_{1DA}(\tau_4, \tau_1; k_\parallel), \quad (39)$$

and again may be put under a form similar to (37):

$$\Gamma_2^{(2)} = \frac{1}{2} \int dx_0 \int dx'_0 q_\perp(x_0) F^{(2)}(x_0 - x'_0) q_\perp(x'_0) \quad (40)$$

where now

$$\begin{aligned} \widetilde{F^{(2)}}(\omega) &= -\frac{1}{8\pi} \int \frac{d^2 k_\parallel}{(2\pi)^2} \int d\nu \frac{1}{c^2(\omega + \nu, k_\parallel) - b^2(\omega + \nu, k_\parallel)} \frac{1}{c^2(\nu, k_\parallel) - b^2(\nu, k_\parallel)} \\ &\times \left[c(\omega + \nu, k_\parallel) c(\nu, k_\parallel) e^{-2a\sqrt{(\omega+\nu)^2 + k_\parallel^2 + m^2}} + b(\omega + \nu, k_\parallel) b(\nu, k_\parallel) e^{-a\sqrt{(\omega+\nu)^2 + k_\parallel^2 + m^2}} e^{-a\sqrt{\nu^2 + k_\parallel^2 + m^2}} \right]. \end{aligned} \quad (41)$$

Thus, the form of Γ_2 shall be:

$$\Gamma_2 = \frac{1}{2} \int dx_0 \int dx'_0 q_\perp(x_0) F(x_0 - x'_0) q_\perp(x'_0) \quad (42)$$

where $F = F^{(1)} + F^{(2)}$.

This is the main result of this section. It gives, to second order in the departure from the normal moving mirror's average position, the Euclidean effective action. At the same order, the *in-out* effective action becomes

$$\Gamma_{2,\text{in-out}}(q_\perp) = \frac{1}{2} \int dx_0 \int dx'_0 q_\perp(x_0) F_{\text{in-out}}(x_0 - x'_0) q_\perp(x'_0), \quad (43)$$

where $F_{\text{in-out}}$ is the continuation of the kernel to Minkowski spacetime, using Feynman's prescription to avoid the poles. Moreover, the dissipative force on the moving mirror is given by

$$F(x_0) = \int dx'_0 F_{\text{ret}}(x_0 - x'_0) q(x'_0), \quad (44)$$

where F_{ret} is the (retarded) continuation to Minkowski spacetime.

C. 1 + 1 dimensions

We will now present some explicit evaluations of the effective action for the 1 + 1 dimensional case. Its formal expression can be obtained in a simple fashion from the previously derived ones. Indeed, beginning from the computation of the kernel $F(x_0 - x'_0)$ in 3 + 1 dimensions one should omit the parallel coordinates $x_{||}$ and the corresponding integration in the $k_{||}$ momenta.

The result for $F^{(1)}$ is

$$\widetilde{F^{(1)}}(\omega) = -\frac{1}{4\pi} \int d\nu \frac{c(\nu + \omega)\sqrt{\nu^2 + m^2}}{c^2(\nu + \omega) - b^2(\nu + \omega)} \quad (45)$$

while for $F^{(2)}$:

$$\widetilde{F^{(2)}}(\omega) = -\frac{1}{8\pi} \int d\nu \frac{c(\omega + \nu)c(\nu)e^{-2a\sqrt{(\omega+\nu)^2+m^2}} + b(\omega + \nu)b(\nu)e^{-a\sqrt{(\omega+\nu)^2+m^2}}e^{-a\sqrt{\nu^2+m^2}}}{(c^2(\omega + \nu) - b^2(\omega + \nu))(c^2(\nu) - b^2(\nu))}. \quad (46)$$

In the particular case of imperfect mirrors such that $\tilde{\lambda}(\omega) = \zeta|\omega|$ (see next section and Appendix A) and $m = 0$, the expressions above simplify to:

$$\widetilde{F^{(1)}}(\omega) = -\frac{1}{2\pi} \int d\nu |\nu + \omega| |\nu| \frac{\chi}{\chi^2 - e^{-2a|\nu+\omega|}} \quad (47)$$

and

$$\widetilde{F^{(2)}}(\omega) = -\frac{1}{2\pi} \int d\nu e^{-2a|\omega+\nu|} \frac{|\omega + \nu||\nu|}{(\chi^2 - e^{-2a|\omega+\nu|})} \frac{(\chi^2 + e^{-2a|\nu|})}{(\chi^2 - e^{-2a|\nu|})}, \quad (48)$$

where we have set $\chi = (2 + \zeta)/\zeta$. The perfect mirror case is obtained by taking the $\zeta \rightarrow \infty$ limit:

$$\widetilde{F^{(1)}}(\omega) \rightarrow \widetilde{F_\infty^{(1)}}(\omega) = -\frac{1}{2\pi} \int d\nu |\nu + \omega| |\nu| \frac{1}{e^{2a|\nu+\omega|} - 1} \quad (49)$$

$$\widetilde{F^{(2)}}(\omega) \rightarrow \widetilde{F_\infty^{(2)}}(\omega) = -\frac{1}{2\pi} \int d\nu e^{-2a|\omega+\nu|} \frac{|\omega + \nu||\nu|}{(1 - e^{-2a|\omega+\nu|})} \frac{(1 + e^{-2a|\nu|})}{(1 - e^{-2a|\nu|})}, \quad (50)$$

where we have subtracted the (already known) contribution corresponding to a single mirror from $\widetilde{F^{(1)}}(\omega)$.

Thus, for perfect mirrors, we have:

$$\begin{aligned} \widetilde{F_\infty}(\omega) &= \widetilde{F_\infty^{(1)}}(\omega) + \widetilde{F_\infty^{(2)}}(\omega) \\ &= \frac{|\omega|^3}{12\pi} - \frac{\omega^2\pi}{6a^3} \left(1 + \frac{\omega^2 a^2}{\pi^2}\right) \sum_{n \geq 1} \frac{1}{\omega^2 + \frac{n^2 \pi^2}{a^2}}. \end{aligned} \quad (51)$$

This form of the kernel is useful to understand its general structure and to perform the analytic continuations. Indeed, following the procedure outlined in Ref.[6], we can use the integral representation

$$|\omega|^3 = \frac{2\omega^4}{\pi} \int_0^{+\infty} dz \frac{1}{\omega^2 + z^2}, \quad (52)$$

so the full kernel is written in terms of the massive 0 + 1 dimensional Euclidean propagator $(\omega^2 + M^2)^{-1}$. Therefore the *in* – *out* kernel for the effective action can be obtained through the substitution $\omega^2 \rightarrow -\omega^2 + i\epsilon$. Moreover, the force on the mirror can be obtained by replacing the Euclidean propagator by the retarded one, i.e. $\omega^2 \rightarrow -(\omega + i\epsilon)^2$. We see that the real time kernels have poles for the particular frequencies $\omega_n = n\pi/a$. These poles indicate the well known breakdown of the perturbative calculation when the mirror oscillates at such resonant frequencies. One can check that the analytic continuations coincide with the results previously found by other authors [9]. In particular, the retarded kernel reads

$$F_{\text{ret}}(x_0) = \frac{\delta'''(x_0)}{12\pi} - \frac{\pi}{6a^2} \theta(t) \sum_{n \geq 0} \delta'(x_0 - 2na) - \frac{1}{6\pi} \theta(t) \sum_{n \geq 0} \delta'''(x_0 - 2na). \quad (53)$$

This analysis can be generalized to arbitrary (bigger than 1) values of χ . The kernels in Eqs. 47 and 48 can be computed analytically. However, the resulting expressions are rather complicated and not very illuminating. In that respect, it is more useful to consider the case of quasi transparent mirrors: $\chi \gg 1$ ($\zeta \ll 1$). The leading terms in an expansion in powers of χ^{-1} are:

$$\begin{aligned}\widetilde{F}^{(1)}(\omega) &= -\frac{1}{2\pi\chi} \int d\nu |\nu + \omega| |\nu| \left(1 + \frac{e^{-2a|\nu+\omega|}}{\chi^2}\right) + \dots \\ \widetilde{F}^{(2)}(\omega) &= -\frac{1}{2\pi\chi^2} \int d\nu |\omega + \nu| |\nu| e^{-2a|\omega+\nu|} + \dots\end{aligned}\quad (54)$$

Therefore:

$$\widetilde{F}(\omega) = -\frac{1}{6\pi\chi} |\omega|^3 - \frac{1}{4\pi a^3 \chi^2} [e^{-2a|\omega|}(1 + a|\omega|) + a|\omega|] + \dots \quad (55)$$

The leading term in the expansion does not depend on a , and equals the kernel of a single imperfect moving mirror. Being proportional to $|w|^3$ (or $\delta'''(x_0)$ in its retarded Fourier transform), it is similar to that of the single perfect moving mirror case. Its form could have been anticipated by using dimensional analysis; indeed, for this particular form of $\tilde{\lambda}(\omega)$, ζ is dimensionless, and therefore one does not have any additional dimensionful constant in the theory.

Using the integral representation

$$|\omega| e^{-2|\omega|a} = \frac{2\omega^2}{\pi} \int_z^\infty dz \frac{e^{2iza}}{\omega^2 + z^2}, \quad (56)$$

one can show that the $1/\chi^2$ -retarded contributions are proportional to $\delta'(x_0)$, $\delta(x_0 - 2a)$, and $\delta'(x_0 - 2a)$. Higher order corrections always involve the δ -function and its derivatives evaluated at $x_0 - 2na$. Once more, this is a characteristic of the specific form used for $\tilde{\lambda}(\omega)$: as the corresponding reflection coefficient has a constant phase, the mirrors do not introduce any delay in a reflecting wave packet. Thus the time of flight between the two mirrors does not depend on χ , and equals the one for perfect mirrors ($\chi = 1$).

IV. SIDEWISE MOTION

Here, the L mirror is again at rest and lying on the $x_3 = 0$ plane, while the R mirror, at a constant distance of the first, is at $x_3 = a$, moves along x_1 : $x_1 = q_{||}(x_0)$.

It is worth to mention that, in this case, one should expect forces only on an *imperfect* mirror. Indeed, there can be no force for a *perfect* mirror, since in such a case the vacuum field will satisfy Dirichlet boundary conditions, irrespective of its state of motion, and the effective action Γ_I becomes proportional to the Casimir energy for two perfect mirrors. However, on an imperfect mirror the boundary conditions will, as we shall see, depend on the state of motion. Therefore, there will be excitations both of the vacuum field and of the microscopic degrees of freedom of the mirror.

Since the L mirror is at rest, the form of its interaction kernel is, simply

$$V_L(x, x') = \delta(x_3) \lambda(x_0 - x'_0) \delta^{(2)}(x_{||} - x'_{||}) \delta(x'_3), \quad (57)$$

while the R has a similar form only in a comoving system:

$$V_R^{(0)}(x, x') = \delta(x_3 - a) \lambda(x_0 - x'_0) \delta^{(2)}(x_{||} - x'_{||}) \delta(x'_3 - a), \quad (58)$$

where the (0) reminds us of the fact that it corresponds to a system where R is at rest. The key point is that, if there is relative motion there is no system where both mirrors are at rest. The interaction is spatially local in the rest frame, when the mirror is moving becomes spatially nonlocal, since it shall link points that are now connected by the evolution of the mirror:

$$\delta(x_1 - x'_1) \rightarrow \delta[x_1 - x'_1 - q_{||}(x_0) + q_{||}(x'_0)]. \quad (59)$$

Thus, the form of the V_R kernel is:

$$V_R(x, x') = \delta(x_3 - a) \lambda(x_0 - x'_0) \delta[x_1 - x'_1 - q_{||}(x_0) + q_{||}(x'_0)] \delta(x_2 - x'_2) \delta(x'_3 - a). \quad (60)$$

In the equations above $q_{||}(x_0)$ describes the rigid translation of the mirror; $v(x_0) = \dot{q}_{||}(x_0)$ will be used for its instantaneous velocity. It is clear that V_R will, in general, acquire a non trivial time dependence, which shall be the origin of the dissipative effects described below.

In what follows, we compute the effective action for different kinds of motions (of the R mirror).

A. Constant velocity (Casimir friction)

As shown several years ago by Pendry [2], two flat surfaces characterized by different dielectric functions experience a friction force when sheared parallel to their interface, even at a constant velocity. This effect is produced by the exchange of virtual photons, that "see" a different reflection coefficient on each surface, due to the non vanishing relative velocity. Recently, there has been some debate about the reality of this effect, see for instance [3]. This in turn has triggered additional works [4] that confirmed the frictional Van der Waals forces between atoms, and atom near a surface, and also between surfaces. We will now show that dissipative effects are also present for the thin mirrors considered in this paper, as long as the effective action for the vacuum field is nonlocal in time.

Note that, in this constant velocity case, it is entirely equivalent to consider Γ_I or Γ , since it is only the relative motion what may produce friction. The effective action corresponding to each isolated mirrors is trivial, as one can guess intuitively, and also by an explicit calculation.

To proceed, we expand the effective action perturbatively in λ . To lowest nontrivial order,

$$\Gamma_I \approx -\frac{1}{2} \text{Tr} \left(\frac{1}{-\partial^2} V_L \frac{1}{-\partial^2} V_R \right), \quad (61)$$

or, in Fourier space,

$$\Gamma_I \approx -\frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \tilde{G}(p) \tilde{V}_L(p, q) \tilde{G}(q) \tilde{V}_R(q, p) \quad (62)$$

where:

$$\begin{aligned} \tilde{G}(p) &= \frac{1}{p^2 + m^2} \\ \tilde{V}_A(p, q) &= \int d^4 x d^4 y e^{-ipx + iqy} V_A(x, y). \end{aligned} \quad (63)$$

For the particular case of constant velocity $\dot{q}_{\parallel} = v$, assuming that the vacuum field is massless, and, as always, that both mirrors are identical at rest,

$$\Gamma \approx \frac{T\Sigma}{64\pi^3} \int d^3 p \frac{e^{-2a\sqrt{p_0^2 + p_1^2 + p_2^2}}}{p_0^2 + p_1^2 + p_2^2} \tilde{\lambda}(p_0) \tilde{\lambda}(p_0 + p_1 v) \quad (64)$$

which is the main result of this section. From it we can derive several interesting conclusions: on the one hand, the derivative of the effective action with respect to a , yields the usual attractive Casimir force between the mirrors, if evaluated perturbatively for 'highly transparent' mirrors.

Equation (64) shows, explicitly, that this force depends on the velocity of the right mirror. This is to be expected, since the motion of the mirror changes its reflection coefficient, and therefore affects its interaction with the vacuum field. On the other hand, in general Γ will have an imaginary part when rotated back from Euclidean to Minkowski spacetime. This is the signal of frictional forces between mirrors. Moreover, one can prove that in order to have friction between the mirrors it is necessary that $\tilde{\lambda}(p_0)$ have a non-analyticity. In other words, the microscopic degrees of freedom in the mirror should induce a non local interaction for the vacuum field. Indeed, when $\tilde{\lambda}(p_0)$ is analytic, then $\lambda(x_0)$ can be written as a linear combination of the delta function and its derivatives, so the interaction S_I can be expanded in local terms involving time-derivatives of the vacuum field. For this particular interaction, the effective action (64) will be analytic in v , and only even powers of v will contribute to the final results. These terms will not produce an imaginary part when the Euclidean velocity is rotated to Minkowski spacetime $v \rightarrow iv$.

As an example to illustrate this point, for a constant velocity, we will assume that $\tilde{\lambda}(p_0) = \zeta |p_0|$. This is a relevant non-analytic interaction since, as shown in Appendix A, it describes a zero-width mirror where the vacuum field propagates with velocity $1/\sqrt{1+\zeta}$; in other words, $1+\zeta$ plays the role of a dielectric constant.

The v -dependent part of the effective action becomes, in this case,

$$\Gamma \approx \frac{T\Sigma\zeta^2}{576\pi^3} \frac{|v|^3}{a^3} + O(v^4), \quad (65)$$

which has a non vanishing imaginary part when continued to Minkowski spacetime.

$$\text{Im}\Gamma_{\text{in-out}} \approx \frac{T\Sigma\zeta^2}{576\pi^3} \frac{|v|^3}{a^3} + O(v^4). \quad (66)$$

B. Non-constant velocity

In this section we compute the form of the effective action for a more general motion of the right mirror. We will follow a similar approach to the one in Section III, and compute the effective action in an expansion in powers of $q_{\parallel}(x_0)$. The difference is, in this case, that q_{\parallel} describes the motion of the R mirror along the x_1 -direction.

We shall assume that the mirror is always close to its equilibrium position, and therefore expand the expression for the V_R in powers of $q(x_0)$; namely, $V_R = V_R^{(0)} + V_R^{(1)} + V_R^{(2)} + \dots$, where:

$$\begin{aligned} V_R^{(0)}(x, x') &= \delta(x_3 - a)\lambda(x_0 - x'_0)\delta^{(2)}(x_{\parallel} - x'_{\parallel})\delta(x'_3 - a) \\ V_R^{(1)}(x, x') &= -(q_{\parallel}(x_0) - q_{\parallel}(x'_0))\lambda(x_0 - x'_0)\delta'(x_1 - x'_1)\delta(x_2 - x'_2)\delta(x_3 - a)\delta(x'_3 - a) \\ V_R^{(2)}(x, x') &= \frac{1}{2}(q_{\parallel}(x_0) - q_{\parallel}(x'_0))^2\lambda(x_0 - x'_0)\delta''(x_1 - x'_1)\delta(x_2 - x'_2)\delta(x_3 - a)\delta(x'_3 - a). \end{aligned} \quad (67)$$

Therefore, there is then an expansion for Γ , $\Gamma_I = \Gamma^{(0)} + \Gamma^{(1)} + \Gamma^{(2)} + \dots$, with

$$\Gamma^{(1)} = \frac{1}{2}\text{Tr}[(-\partial^2 + V_L + V_R^{(0)})^{-1}V_R^{(1)}] \quad (68)$$

and

$$\begin{aligned} \Gamma^{(2)} &= \frac{1}{2}\text{Tr}[(-\partial^2 + V_L + V_R^{(0)})^{-1}V_R^{(2)}] - \frac{1}{4}\text{Tr}\left[\left((-\partial^2 + V_L + V_R^{(0)})^{-1}V_R^{(1)}\right)^2\right] \\ &\equiv \Gamma^{(2,1)} + \Gamma^{(2,2)}. \end{aligned} \quad (69)$$

It is straightforward to see that $\Gamma^{(1)} = 0$; that is, that there is no linear term in the velocity. Introducing the notation

$$\mathcal{G} = (-\partial^2 + V_L + V_R^{(0)})^{-1} \quad (70)$$

and denoting by $\tilde{\mathcal{G}}$ its Fourier transform, one sees that

$$\Gamma^{(2,1)} = \Sigma \int dk_0 \tilde{q}(-k_0) \tilde{\Pi}^{(2,1)}(k_0) \tilde{q}(k_0) \quad (71)$$

where

$$\tilde{\Pi}^{(2,1)}(k_0) = \frac{1}{32\pi^4} \int d\omega dk_1 dk_2 k_1^2 \tilde{\lambda}(\omega - k_0) \tilde{\mathcal{G}}(\omega, k_1, k_2, a, a). \quad (72)$$

The evaluation of $\Gamma^{(2,2)}$ is straightforward, although algebraically more involved. The final result can be written as

$$\Gamma^{(2,2)} = \Sigma \int dk_0 \tilde{q}(k_0) \tilde{q}(-k_0) \tilde{\Pi}^{(2,2)}(k_0), \quad (73)$$

where

$$\begin{aligned} \tilde{\Pi}^{(2,2)}(k_0) &= \frac{1}{64\pi^4} \int dl_0 \left[2\tilde{\lambda}(l_0)\tilde{\lambda}(k_0 + l_0) - \tilde{\lambda}(k_0 + l_0)\tilde{\lambda}(k_0 + l_0) - \tilde{\lambda}(l_0)\tilde{\lambda}(l_0) \right] \\ &\times \int dk_1 dk_2 k_1^2 \tilde{\mathcal{G}}(k_0 + l_0, k_1, k_2, a, a) \tilde{\mathcal{G}}(l_0, k_1, k_2, a, a). \end{aligned} \quad (74)$$

Finally, we note that, as at the end of Section III B, the force on the mirror and the imaginary part of the *in-out* effective action can be obtained by appropriate continuations to Minkowski spacetime of the Euclidean form factor $\Pi^{(2,1)} + \Pi^{(2,2)}$. Moreover, there are dissipative effects even for a single mirror sliding with non-vanishing acceleration [10].

V. CONCLUSIONS

In this paper we have presented a functional approach to compute the dissipative effects on imperfect moving mirrors. A crucial point in our approach is that the interaction of the vacuum field with the mirror is modeled by a *nonlocal* term in the action of the quantum field. This kind of term is generated by the interaction of the field with the internal degrees of freedom of the mirror, in the lowest non trivial order in an expansion in powers of the field.

The consideration of the quantum theory associated to the degrees of freedom of the *whole* system (vacuum field and mirrors) allowed us to compute both the dissipative effects coming from the excitation of the vacuum field, and those coming from the excitation of the internal degrees of freedom of the mirror. Moreover, we have seen that it is particularly convenient to work in imaginary time (Euclidean spacetime), in order to obtain the probability of excitation of the system and the dissipative force on the moving mirror from adequate analytic continuations of the effective action to Minkowski spacetime.

For the sake of simplicity, we have considered a simplified model with a quantum scalar field, coupled to flat thin mirrors which undergo rigid motion. We have also assumed that the interaction term is nonlocal in time, and local in the spatial coordinates of the mirror, as it happens when the internal degrees of freedom can be described by a set of independent harmonic oscillators. Under these assumptions, we have been able to analyze several interesting situations that involve two mirrors in normal or parallel relative motion. In the first case, we computed the effective action of the system for a moving mirror near a static mirror, perturbatively in the departure from the equilibrium position. In the second case, we have shown that there is a dissipative force between the mirrors when the sliding motion has constant velocity. We have seen that the temporal nonlocality is a necessary condition for non-vanishing friction. Moreover, using again a perturbative approach, we have obtained a general expression for the effective action as a functional of the position of the mirror. As a particular case, we pointed out that there is dissipation even for a single mirror with sliding accelerated motion.

The approach described here can be generalized in several directions. On the one hand, one should consider the more realistic case of the electromagnetic field. On the other, one could consider more general nonlocal interactions, induced by internal degrees of freedom that propagate inside the mirror. For instance, it would be interesting to analyze the dynamical Casimir effect for graphene sheets. In this case, the internal degrees of freedom can be described with massless Dirac fields [11], propagating at a velocity $v_F \ll c$ inside the mirror, and will produce an effective interaction that is both temporally and spatially nonlocal.

Acknowledgements

C.D.F. thanks CONICET, ANPCyT and UNCuyo for financial support. The work of F.D.M. and F.C.L was supported by UBA, CONICET and ANPCyT.

Appendix A

In this Appendix we show that, when the interaction between the internal degrees of freedom of a zero-width mirror and the vacuum field is described by the non-analytic function $\tilde{\lambda}(p_0) = \zeta|p_0|$, the vacuum field propagates with velocity $1/\sqrt{1+\zeta}$ on the mirror.

We consider then a mirror static at $x_3 = 0$, and to study the propagation of φ modes on that plane, we add a coupling to an external source J , which is localized on the mirror: $J(x) = j(x_0, x_1, x_2)\delta(x_3)$.

Then, by integrating out the vacuum field in the resulting theory, one finds the generating functional:

$$\mathcal{Z}(j) = \int \mathcal{D}\varphi e^{-S(\varphi) + i \int d^4x J(x)\varphi(x)} \quad (75)$$

whence we can obtain the propagator for φ exactly on the plate, by taking derivatives of $\mathcal{Z}(j)$ with respect to j .

The integral over the vacuum field is of course Gaussian, and its result is

$$\langle \varphi(x_0, x_1, x_2) \varphi(x'_0, x'_1, x'_2) \rangle = \int \frac{d\omega}{2\pi} \frac{d^2 k_{\parallel}}{(2\pi)^2} \frac{e^{i\omega(x_0 - x'_0) + i k_{\parallel} \cdot (x_{\parallel} - x'_{\parallel})}}{2(|k_{\parallel}| + \zeta|\omega|)}. \quad (76)$$

Propagating modes on the plate are then to be found as the singularities in the real time version of the propagator in momentum space. They are then solutions to the equation:

$$i\zeta|\omega| + i\sqrt{-\omega^2 + \mathbf{k}_{\parallel}^2} = 0, \quad (77)$$

Those solutions are: $\omega = \pm v |\mathbf{k}_{\parallel}|$, where $v \equiv 1/\sqrt{1+\zeta}$. They are modes with $v < c$, for $\zeta > 0$. Thus, a natural interpretation of ζ , in this context, is that it represents (in an EM analogy) a plate where $\epsilon > 1$, where $1 + \zeta = \epsilon$.

A simpler derivation can be obtained by studying the classical equations of motion for the vacuum field, in the presence of the mirror at $x_3 = 0$. Fourier transforming the equation with respect to the time and x_{\parallel} , we see that the Fourier transformed vacuum field should satisfy:

$$[\partial_3^2 - k_{\parallel}^2 + \omega^2 - \tilde{\lambda}(\omega)\delta(x_3)] \tilde{\varphi}(x_3, \omega, k_{\parallel}) = 0. \quad (78)$$

Thus, nontrivial solutions for $\tilde{\varphi}(x_3, \omega, k_{\parallel})$ must be of the form:

$$\tilde{\varphi}(x_3, \omega, k_{\parallel}) = f(\omega, k_{\parallel}) e^{-\sqrt{k_{\parallel}^2 - \omega^2} |x_3|}, \quad (79)$$

which, in order to satisfy the discontinuity imposed by the δ function, requires:

$$2\sqrt{k_{\parallel}^2 - \omega^2} = \tilde{\lambda}(\omega), \quad (80)$$

which has the same solutions that (77).

Note that there are also solution which are odd in x_3 , and therefore they do not see the δ function. However, since the vacuum field vanishes on the plane, they do not propagate modes on the mirror.

-
- [1] V.V. Dodonov in *Modern Nonlinear Optics*, Advances in Chemical Physics Series, ed. by M.W. Evans (Wiley, New York 2001), Vol. 119, p. 309; A. Lambrecht, J. Opt. B: Quantum Semiclass. Opt. **7**, S3 (2005); V.V. Dodonov, J. Phys.: Conf. Ser. **161**, 012027 (2009); D. A. R. Dalvit, P. A. Maia Neto and F. D. Mazzitelli, arXiv:1006.4790, to appear in Lecture Notes in Physics, Volume on Casimir Physics, ed. by D.A.R. Dalvit, P. Milonni, D. Roberts, and F. da Rosa.
 - [2] J.B. Pendry, J. Phys.:Condens. Matter **9**, 10301 (1997).
 - [3] J.B. Pendry, New J. Phys. **12**, 033028 (2010); *ibidem* New J. Phys. **12**, 068002 (2010); T.G. Philbin and U. Leonhardt, New J. Phys. **11**, 033035 (2009); U. Leonhardt, New J. Phys. **12**, 068001 (2010).
 - [4] J.S. Hoyer and Brevik I., EPL, **91**, 60003 (2010); *ibidem* Europ. Phys. J. D, in press (arXiv:1009.3135 v2); G. Barton, New J. Phys. **12**, 113044 (2010); *ibidem* New J. Phys. **12**, 113045 (2010).
 - [5] C. D. Fosco, F. C. Lombardo and F. D. Mazzitelli, Phys. Rev. D **82** (2010) 125039 [arXiv:1011.0463 [hep-th]].
 - [6] C. D. Fosco, F. C. Lombardo and F. D. Mazzitelli, Phys. Rev. D **76**, 085007 (2007) [arXiv:0705.2960 [hep-th]].
 - [7] C. D. Fosco, F. C. Lombardo and F. D. Mazzitelli, Phys. Lett. B **669** (2008) 371 [arXiv:0807.3539 [hep-th]].
 - [8] R. Golestanian and M. Kardar, Phys. Rev. A **58**, 1713 (1998).
 - [9] See D. F. Mundarain and P. A. Maia Neto, Phys. Rev. A **57** (1998) 1379 [arXiv:quant-ph/9808064], and references therein.
 - [10] G. Barton, Annals of Physics (New York), **245**, 361(1996); A. Manjavacas and F.J. Garcia de Abajo, Phys. Rev. Lett. **105**, 113601 (2010).
 - [11] M.I. Katsnelson and K.S. Novoselov, Solid State Comm. **143**, 3 (2007).